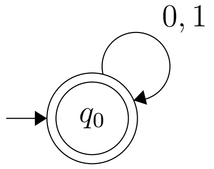
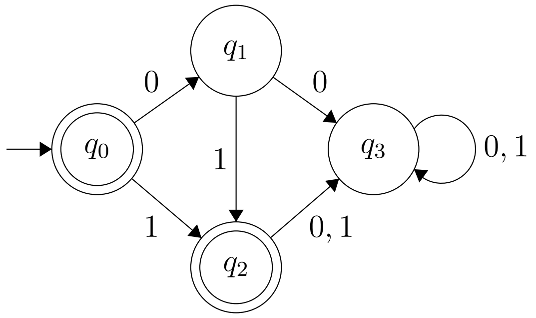
Solution sketch 1 - Computational Models - Spring 2017

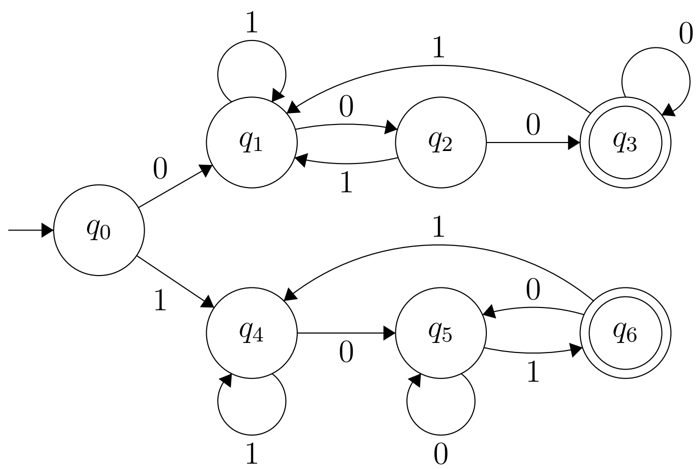
1. (a) Σ^*



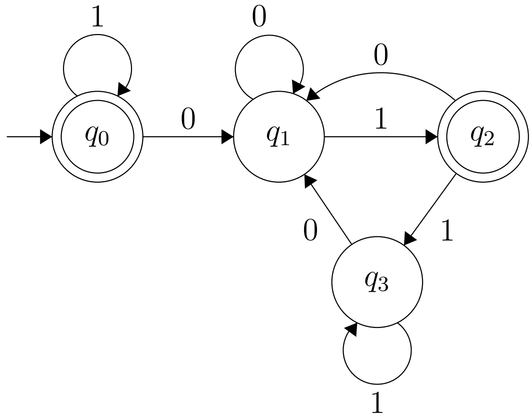
(b) $\{\varepsilon, 1, 01\}$



(c) $\{\sigma w 0 \sigma \mid \sigma \in \Sigma, w \in \Sigma^*\}$



(d) $\{w \mid w \text{ does not contain } 0 \text{ or } w \text{ ends with } 01\}$



2. DFA: $A = (Q, \Sigma, \delta, q_0, F)$. $Q = \{x_1 x_2 \cdots x_n \mid \forall i. x_i \in \{0,1\}\}$, $\Sigma = \{0,1\}$, $q_0 = 1^n$, $F = \{x_1 x_2 \cdots x_n \mid x_1 = 0\}$, $\forall q \in Q, \sigma \in \Sigma, \delta(q,\sigma) = \delta(x_1 x_2 \cdots x_n, \sigma) = x_2 \cdots x_n \sigma$.

3. (a) False. for $L = \Sigma^*$, $\epsilon \in (L \backslash \{\epsilon\})^*$ but $\epsilon \not\in= L^* \backslash \{\epsilon\}$

   (b) True. Since $L^* = \cup_{k \geq 0} L^k$, then for every language A, $A \subseteq A^*$ and thus $L^* \subseteq (L^*)^*$. In the other direction - Let $w \in (L^*)^*$. Thus $w = x_1, ... x_k$ such that $\forall i, x_i \in L^*$. this mean that $\forall i$, $x_i = y_1, ... y_m$ such that $\forall i, y_i \in L$ therefore $w \in L^*$ and $(L^*)^* \subseteq L^*$

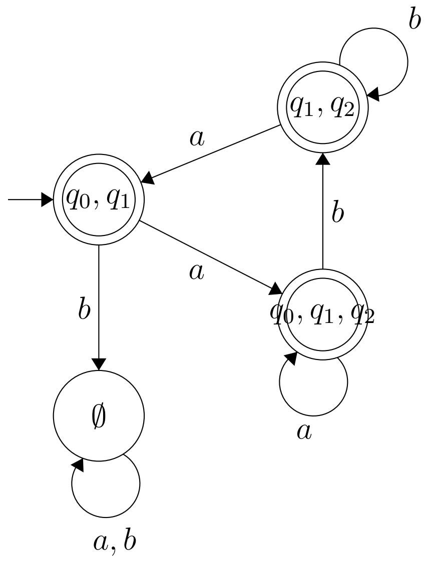
   (c) True. $L_1 \cup L_2 \subseteq L_1^* \cdot L_2^*$ and thus, $(L_1 \cup L_2)^* \subseteq (L_1^* \cdot L_2^*)^*$.
   Let $w \in (L_1^* \cdot L_2^*)^*$. Thus $w = x_1, ... x_k$ such that $\forall i, x_i \in L_1^* \cdot L_2^*$. this mean that $\forall i$, $x_i = y_1, ... y_m, z_1, ... z_n$ such that $\forall i, y_i \in L_1$ and $z_i \in L_2$ therefore $w \in (L_1 \cup L_2)^*$ and $(L_1^* \cdot L_2^*)^* \subseteq (L_1 \cup L_2)^*$

   (d) True. Since $\epsilon \in \Sigma^*$, $\Sigma^* \subseteq \Sigma^* \cdot \Sigma^*$. in the other direction since $\Sigma^* \cdot \Sigma^*$ is over Σ and Σ^* contains all the words over Σ then $\Sigma^* \cdot \Sigma^* \subseteq \Sigma^*$

4. (a) $M = (Q = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}, \delta, q_0, \{q_1\})$, δ :

	a	b	ε
q_0	$\{q_2\}$	$\emptyset$	$\{q_1\}$
q_1	$\{q_0\}$	$\emptyset$	$\emptyset$
q_2	$\{q_1\}$	$\{q_1, q_2\}$	$\emptyset$

   (b) The equivalent DFA



5. (a) $1^*01^*01^*01^*01^*$

   (b) $(0 \cup 1)^*000(0 \cup 1)^*$

6. Can be proved using the claim:

   Claim. *For any* $w \in \Sigma^*$ *and* $q \in Q$ *it holds that if* $q \in \widehat{\delta}(S, w) \Rightarrow \exists a = (a_1 a_2 \dots a_k) \in (\Sigma_\varepsilon)^k$ *and* $r_0, \dots, r_k \in Q$ *s.t.,*

   - $w = d(a)$
   - $r_0 \in S$
   - $r_k = q$
   - $r_{i+1} \in \delta(r_i, a_{i+1})$, *for all* $0 \leq i < k$

   The above claim can proved by induction on word length.

7. This language is $L\bar{L} \cup \bar{L}L$. Since L is regular and the regular languages are closed under the operators complement, union and concatenation, the language at hand is also regular.