Exercise 6 - Computational Models - Spring 2017

1. Show that the following languages are in P
   (a) $SDNF = \{\langle\varphi\rangle \mid \varphi \text{ is a satisfiable DNF formula}\}$
   (b) $TCNF = \{\langle\varphi\rangle \mid \varphi \text{ is a CNF tautology}\}$

2. Prove:
   (a) If $L \in NP$ then $L^* \in NP$.
   (b) If $L \in P$ then $L^* \in P$.

3. We say that a polynomial reduction f is a *shrinking reduction* if there exists n_0 such that for every $x \in \Sigma^*$ such that $|x| \geq n_0$, $|f(x)| \leq |x| - 1$. Assuming $P \neq NP$, prove/disprove:
   (a) For every two nontrivial languages $A, B \in P$ there exists a shrinking reduction from A to B.
   (b) For every two nontrivial languages $A, B \in NPC$ there exists a shrinking reduction from A to B.

4. Let

   $UpToOneSAT = \{\langle\varphi\rangle \mid \varphi \text{ is a Boolean formula that has at most one satisfying assignment}\}$

   Prove that $UpToOneSAT \in NP$ implies $NP = coNP$.

   Hint: Show that $UpToOneSAT \in$ coNP and give a reduction $SAT \leq_p \overline{UpToOneSAT}$.

5. Let

   $HALF\text{-}CLIQUE = \{\langle G\rangle \mid G = (V, E) \text{ is an undirected graph having a clique with at least } \frac{|V|}{2} \text{ nodes}\}$

   Show that *HALF-CLIQUE* is NP-complete.

6. Given an undirected graph $G = (V, E)$, a *vertex cover* of a G is a set $S \subseteq V$ such that $\forall\{u, v\} \in E$, $u \in S$ or $v \in S$ (or both). Namely, every edge has at least one endpoint in S. Let

   $$VC = \{\langle G, k\rangle \mid G \text{ is an undirected graph with a vertex cover of size } k\}$$

   Show that VC is NP-complete.