

Exercise 5 - Computational Models - Spring 2017

1. Let A , B and C languages over Σ . Prove/disprove:
 - (a) If $A \leq_m B$ and B is a regular language, then A is a regular language.
 - (b) If A is not trivial and A context-free, then $A \leq_m A_{TM}$.
 - (c) If $A \leq_m B$ and $B \leq_m C$, then $A \leq_m C$.
 - (d) For every A and B , either $A \leq_m B$ or $B \leq_m A$.
2. A *useless* state in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a state in a Turing machine is useless:

$$USELESS_{TM} = \{\langle M, q \rangle \mid q \text{ is a useless state in the TM } M\}$$

Show that $USELESS_{TM}$ is undecidable.

3. Let $S_{TM} = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that S_{TM} is undecidable.
4. For the following decision problems, determine whether they belong to R , $RE \setminus R$, $coRE \setminus R$ or outside of $RE \cup coRE$. Prove your answer.
 - (a) $L = \{\langle M \rangle \mid M \text{ is a TM and } A_{TM} \leq_M L(M)\}$.
 - (b) $L = \{\langle M \rangle \mid M \text{ is a TM and for every } x, M\text{'s run on } x \text{ never reaches position } |x| + 20\}$.
5. Assume that we work with a one-tape TM, with $\Sigma = \{0, 1\}$ and $\Gamma = \{0, 1, \sqcup\}$. Let f be a function that given $\langle M \rangle$ return the index of rightmost cell visited by M when running on ε . If there is no such index $f(\langle M \rangle) = \infty$. Show that f is not computable.
6. Let x be a string. We say that x is *incompressible* if $K_U(x) \geq |x|$. Let $S = \{x \in \{0, 1\}^* \mid K_U(x) \geq |x|\}$ (i.e. the set of incompressible strings). Show that the set of incompressible strings contains no infinite subset that is recursively enumerable, namely show that there is no infinite set $A \subseteq S$ s.t. $A \in RE$.