Computational Models - Exercise #4, Spring 2017

Due: June 9, 14:00

1. A TM computes a (maybe partial) function $f : \Sigma^\star \to \Gamma^\star$ if for every input x in the domain of f, it halts with $f(x)$ written to the left of the head. Give a formal description (specifically, of the δ function itself) of a TM that computes the function $f(x) = x + 1$, where x is given in binary from left to right (e.g., for $x = 6$ the tape's content is 110 and the head first reads the MSB, 1).

   No formal proof of correctness is needed, but do provide a detailed explanation.

   <u>Not for submission</u>: Start writing the binary string that is the encoding of your TM. Also, make sure that you are capable of writing an algorithm that on input $w \in \{0,1\}^\star$, outputs whether w is a legal encoding of some TM.

2. Let $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be a PDA. Give a formal definition of a TM M so that $L(M) = L(P)$. Give a detailed explanation of your construction. You can use any equivalent model we saw in class.

3. We define an alternative model of computation in which the TM can alter each tape cell at most once (including the input portion of the tape).

   Is the new model of computation equivalent to the standard one? If so, give a detailed explanation. If not, present a specific language that demonstrates it. A partial (but generous) credit will be given for solving the question with "at most twice" instead of once.

4. For languages L_1, L_2 over some alphabet Σ, we define

   $$Shuffle(L_1, L_2) = \{a_1 b_1 a_2 b_2 \cdots a_n b_n : n \geq 1 \wedge a_1, \ldots, a_n \in L_1 \setminus \{\varepsilon\} \wedge b_1, \ldots, b_n \in L_2 \setminus \{\varepsilon\}\}$$

   (a) Prove that if $L_1, L_2 \in \mathcal{RE}$ then $Suffle(L_1, L_2) \in \mathcal{RE}$.

   (b) (<u>not for submission</u>) Does the above hold for co$\mathcal{RE}$?

5. Prove/disprove:

   (a) If $L_1, L_2 \in \mathcal{RE} \setminus \mathcal{R}$ then $L_1 \cap L_2 \notin \mathcal{R}$.

   (b) co$\mathcal{RE}$ is closed under intersection.

   (c) $\overline{\mathcal{R}}$ is closed under union.

   (d) $\mathcal{RE}$ is closed under homomorphisms.

6. Let M be a one-tape TM that visits every cell at most k times, for some fixed $k \in \mathbb{N}$. Prove that $L(M)$ is regular.

Hint: Use crossing sequences.[1] What does our constraint on M say on the crossing sequences? How can we use it?

This exercise is challenging, so partial results will also be generously graded.

[1] A *crossing sequence* at location i on input x, is the sequence of states M is in when its head crosses the boundary between the i-th cell and the $(i+1)$-th cell, when the TM is run on input x.