Exercise 1 - Computational Models - Spring 2017

1. For each of the following languages over $\Sigma = \{0, 1\}$, present a drawing representing a DFA that accepts it (correctness proof not needed):

   (a) Σ^*

   (b) $\{\varepsilon, 1, 01\}$

   (c) $\{\sigma w 0 \sigma \mid \sigma \in \Sigma, w \in \Sigma^*\}$

   (d) $\{w \mid w \textit{ does not contain } 0 \textit{ or } w \textit{ ends with } 01\}$

2. Given n, let L_n be the language of words over $\Sigma = \{0, 1\}$ such that the nth character from the end is 0. Present a DFA that accepts L_n (the answer should be depend on n). Give a formal description and not a drawing.

3. Let L, L_1, L_2 a languages over Σ. Prove/Disprove the following statements:

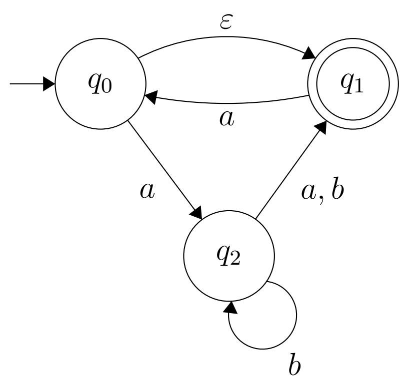
   (a) $(L \backslash \{\epsilon\})^* = L^* \backslash \{\epsilon\}$

   (b) $L^* = (L^*)^*$

   (c) $(L_1 \cup L_2)^* = (L_1^* \cdot L_2^*)^*$

   (d) $\Sigma^* = \Sigma^* \cdot \Sigma^*$

4. (a) Give a formal description of the following NFA:



   (b) Convert the above NFA to an equivalent DFA.

5. Present a regular expression for the following languages over $\Sigma = \{0, 1\}$:

   (a) $\{w|\ w$ *contains exactly four '0's*$\}$

   (b) The complement of $\mathcal{L}((1 \cup 01 \cup 001)^*(\varepsilon \cup 0 \cup 00))$

6. The following question deals with the equivalence between the two definitions for an NFA accepting a string given in class

   Definition 1 $N = (Q, \Sigma, \delta, S, F)$ *accepts* $w \in \Sigma^*$ *if* $\widehat{\delta}_N(S, w) \cap F \neq \emptyset$.

   Definition 2 $N = (Q, \Sigma, \delta, S, F)$ *accepts* $w \in \Sigma^*$, *if* $\exists a = (a_1 a_2 \ldots a_k) \in (\Sigma_\varepsilon)^k$ *and* $r_0, \ldots, r_k \in Q$ *s.t.,*

   - $w = d(a)$[1]
   - $r_0 \in S$
   - $r_k \in F$
   - $r_{i+1} \in \delta(r_i, a_{i+1})$, *for all* $0 \leq i < k$

   Prove that if an NFA accepts a string according to definition 1 then it also does so according to definition 2

7. Given that L is a regular language over some alphabet Σ, prove that the following language is regular: $\{xy|\ (x \in L)\ XOR\ (y \in L)\}$

[1] For $a = (a_1 a_2 \ldots a_k) \in (\Sigma_\varepsilon)^k$, $d(a)$ is a without the ε symbols